CIVIL SERVICE EMPLOYEES ASSOCIATION
LEAP
LABOR EDUCATION ACTION PROGRAM

Examination Preparation Booklet

Basic Mathematics

Booklet No. 1

CSEA
CIVIL SERVICE EMPLOYEES ASSOCIATION, INC.
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Booklet #1

Basic Mathematics

The CSEA Examination Preparation Booklet Series is designed to help members prepare for New York State and local government civil service examinations. This booklet is designed for practice purposes only and its content may not conform to that of any particular civil service examination.

BASIC MATHEMATICS

This booklet is designed to refresh your understanding of basic mathematical operations involving fractions, decimals, percents and ratios. Knowledge of these is required for performing well on the math sections of promotional examinations. This booklet is a refresher of the operations themselves.

Booklets 2 and 3, Arithmetic Reasoning and Understanding and Interpreting Tabular Material, offer an extensive review of the kinds of questions asked on examinations. We suggest you use those booklets as well when an exam you're taking has either Arithmetic Reasoning or Tabular material on it.

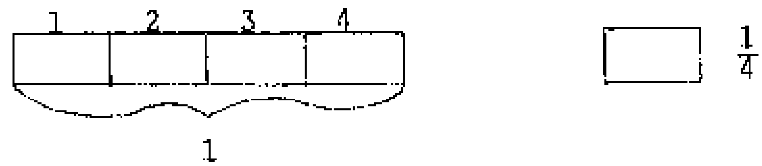
Good luck!

FRACTIONS

I. Definition: A <u>fraction</u> represents part of a whole.

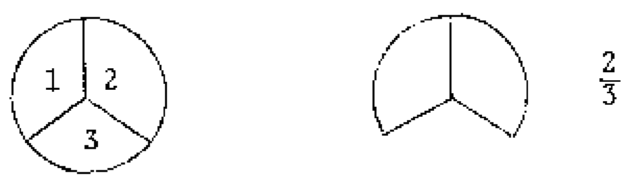
example:

a. $\frac{1}{4}$ means dividing one into 4 pieces, and taking one of them.



b. $\frac{1}{3}$ means dividing one into 3 pieces, and taking one of them.

c. $\frac{2}{3}$ means dividing one into 3 pieces, and taking 2 of them.



Terms: The <u>numerator</u> is the top number of the fraction.
The <u>denominator</u> is the bottom number of the fraction.

Any whole number (1, 2, 3, etc.) can be written as a fraction with a denominator of 1. $\frac{2}{1}$ means 2 wholes, which have not been divided into smaller units.

$\frac{5}{1} = 5$ $\qquad\qquad$ $\frac{210}{1} = 210$

One can be represented by any fraction which has the numerator and denominator equal to the same number.

$1 = \frac{3}{3}$ $\qquad\qquad$ $1 = \frac{5}{5}$

II. Multiplication

Multiply the numerators together and multiply the denominators together.

<u>Symbols</u> for multiplication are x, ·, ( ), ( )

example:

a. $\frac{3}{4} \times \frac{2}{5} = \frac{3}{4} \cdot \frac{2}{5} = \left(\frac{3}{4}\right)\left(\frac{2}{5}\right)$

$\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \frac{6}{20}$

b. $\frac{5}{7} \times \frac{2}{3} = \frac{5 \times 2}{7 \times 3} = \frac{10}{21}$

Multiplication of fractions can be simplified by <u>cancelling</u> or <u>reducing</u> the fractions involved. Cancelling is reducing a fraction to its simplest terms. This is done by finding numbers which divide evenly into both the numerator and denominator.

example: $\frac{3}{6}$

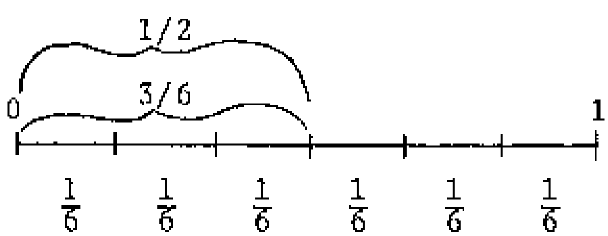
Both the top and bottom of this fraction can be divided evenly by 3.

$3 \div 3 = 1$ $6 \div 3 = 2$

$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$

As long as the top and bottom are both divided by the same number, the value of the fraction remains the same.

We can see what has happened by using a number line:



When multiplying fractions we can also cancel the numerator of one fraction with the denominator of another, if both are divisible by a common number.

example: $\frac{3}{5} \times \frac{2}{9} = ?$

In this problem both fractions are in simplest terms, but the numerator of the first (3) and the denominator of the second (9) can both be divided by 3.

$$\frac{3 \div 3}{5} \times \frac{2}{9 \div 3} = \frac{1}{5} \times \frac{2}{3} = \frac{1 \times 2}{5 \times 3} = \frac{2}{15}$$

This can be written more simply as follows:

$$\frac{\cancel{3}\,1}{5} \times \frac{2}{\cancel{9}\,3} = \frac{2}{15}$$

When working with fractions (adding, subtracting, multiplying or dividing) final answers should always be reduced to lowest (simplest) terms.

Clues for reducing or cancelling fractions:

1. When the numerator and denominator both end in zero (0), they can be reduced by 10.

$$\frac{150}{40} = \frac{150 \div 10}{40 \div 10} = \frac{15}{4} \quad \text{or} \quad \frac{15\cancel{0}}{4\cancel{0}} = \frac{15}{4}$$

2. If both are even, they can be reduced by 2.

$$\frac{8}{26} = \frac{4}{13}$$

3. If both end in either a zero (0) or a 5, the fraction can be reduced by 5.

$$\frac{15}{20} = \frac{15 \div 5}{20 \div 5} = \frac{3}{4}$$

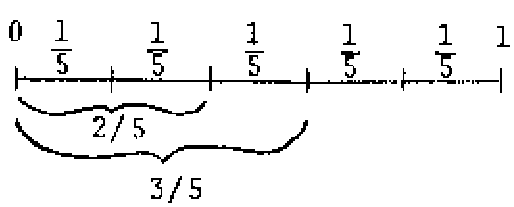
$$\frac{25}{35} = \frac{25 \div 5}{35 \div 5} = \frac{5}{7}$$

III. Addition

When adding fractions we can only add together parts of the same "size," meaning, those fractions with the same denominator. $\frac{1}{4}$ can be added directly to $\frac{3}{4}$, but not directly to $\frac{2}{3}$.

Fractions with the same denominator are added together by adding <u>only</u> the numerators and maintaining the same denominator.

example:



$$\frac{1}{5} + \frac{1}{5} = \frac{2}{5} \qquad\qquad \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

What if the fractions do not have the same denominator? In this case, we convert them into new fractions which do have the same denominator. This denominator is called a <u>common denominator</u>.

A <u>common denominator</u> is a number both original denominators can be <u>divided into evenly</u>. A common denominator can be found for all fractional combinations.

1. One way to find a common denominator is to multiply the original denominators together.

example: $\frac{1}{3} + \frac{2}{5} = ?$

$3 \times 5 = 15$

Therefore, 15 would be the common denominator.

When converting the original fraction into a fraction with a common denominator, you must be sure to keep the value of the fraction the same. To do this, multiply both the numerator and denominator by the same number. Using the example above you proceed as follows:

$$\frac{1 \times 5}{3 \times 5} = \frac{5}{15} \qquad \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

When you multiply both the top and bottom by the same number the value of the fraction does not change, because you are really multiplying by one. This problem can be finished by adding the numerators together, and maintaining the common denominator.

$$\frac{5}{15} + \frac{6}{15} = \frac{5+6}{15} = \frac{11}{15}$$

2. A smaller common denominator can often be found by mentally reviewing multiplication tables to find the smallest number both denominators divide into evenly.

example: $\frac{1}{6} + \frac{3}{8} = ?$

Both 6 and 8 will divide evenly into 24.

$24 \div 6 = 4 \qquad 24 \div 8 = 3$

Each denominator is multiplied by the number which will make it equal 24. The numerators are also multiplied by this number, so that the value of the fractions isn't changed.

$$\frac{1 \times 4}{6 \times 4} = \frac{4}{24} \qquad \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

$$\frac{4}{24} + \frac{9}{24} = \frac{4+9}{24} = \frac{13}{24}$$

Using the last method (multiplying denominators) we would proceed as follows:

$$\frac{1\times8}{6\times8} = \frac{8}{48} \qquad \frac{3\times6}{8\times6} = \frac{18}{48}$$

$$\frac{8}{48} + \frac{18}{48} = \frac{8+18}{48} = \frac{26}{48}$$

$\frac{26}{48}$ can be reduced to $\frac{13}{24}$ by cancelling.

IV. Subtraction

Subtracting fractions is similar to adding fractions. First all of the fractions in the problem must be converted into new fractions with a common denominator. Once both fractions have a common denominator, the second numerator is subtracted from the first.

example: $\frac{3}{4} - \frac{1}{6} = ?$

The common denominator can be found 2 ways:

1. Multiply the denominators.

$4 \times 6 = 24$, the common denominator

$$\frac{3}{4} \times \frac{6}{6} = \frac{18}{24} \qquad \frac{1}{6} \times \frac{4}{4} = \frac{4}{24}$$

$$\frac{18}{24} - \frac{4}{24} = \frac{18-4}{24} = \frac{14}{24}$$

$\frac{14}{24}$ can be reduced to lower terms, or $\frac{7}{12}$.

2. Finding a smaller common denominator by finding a number both denominators will divide into evenly.

4 and 6 will both divide evenly into 12. 12 is smaller than 24, the denominator used in part 1. Since $12 \div 4 = 3$ and $12 \div 6 = 2$ we convert these fractions as follows:

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \qquad \frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$$

$$\frac{9}{12} - \frac{2}{12} = \frac{9-2}{12} = \frac{7}{12}$$

V. Division

In a division problem, the <u>divisor</u> is the number we are dividing by. It is written after the ÷ sign, or on the bottom of the slash.

example: $\frac{2}{3} \div \frac{1}{4}$ — $\frac{1}{4}$ is the divisor

$\frac{\frac{5}{8}}{\frac{1}{2}} = \frac{5}{8} \div \frac{1}{2}$ — $\frac{1}{2}$ is the divisor

Fractions are divided by inverting the divisor, and then multiplying. To invert the divisor, switch the numerator and denominator. Be sure to switch the division sign to a multiplication sign. Once the divisor has been inverted, multiply the 2 fractions together to get the answer.

example: $\frac{2}{3} \div \left(\frac{1}{4}\right) = ?$

$$\frac{2}{3} \times \frac{4}{1} = \frac{2\times4}{3\times1} = \frac{8}{3}$$

Example: $\frac{\frac{5}{8}}{\frac{1}{2}} = ?$

$$\frac{5}{8} \div \left(\frac{1}{2}\right) = \frac{5}{8} \times \frac{2}{1} = \frac{5\times2}{8\times1} = \frac{\not{1}\not{0}\,5}{\not{8}\,4} = \frac{5}{4}$$

VI. Mixed Numbers

A <u>mixed number</u> has a whole number part and a fractional part (for example: $1\frac{1}{2}$, $20\frac{5}{9}$).

A mixed number can be represented entirely as a fraction. The new fraction will have the same denominator as the fractional part of the mixed number.

example: $2\frac{3}{4} = \frac{?}{4}$

Begin by looking at the whole number, $2 = \frac{?}{4}$. To find the numerator of this part, multiply the whole number by the denominator.

$$2 \times 4 = 8$$

Therefore, 2 represented in fractional form equals $\frac{8}{4}$.

$$2 = \frac{8}{4}$$

Now, add this to the fractional part of the mixed number.

$$2\,\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}$$

Fractions with the numerator greater than the denominator can be converted to mixed numbers. This is done by dividing the denominator into the numerator. The number of times the denominator goes into the numerator evenly is the whole number part of the mixed number. The remainder (the amount left over after dividing) is the new numerator of the fractional part. The denominator is the same.

example: $\frac{14}{3} = ?$

Three goes into 14 evenly four times. The remainder is 2.

$$3\,\overline{)14}\;=4,\quad 14-12=2$$

$$\frac{14}{3} = 4\,\frac{2}{3}$$

VII. Multiplying Mixed Numbers and Fractions

example: $3\,\frac{1}{4} \times \frac{5}{8} = ?$

When multiplying with mixed numbers, first change the mixed numbers to a fraction.

$$3\,\frac{1}{4} = \frac{?}{4} \qquad 3 \times 4 = 12 \qquad 3 = \frac{12}{4}$$

$$3\,\frac{1}{4} = 3 + \frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{12+1}{4} = \frac{13}{4}$$

Once the mixed number has been converted to a fraction, this becomes a regular multiplication problem.

$$\frac{13}{4} \times \frac{5}{8} = \frac{13 \times 5}{4 \times 8} = \frac{65}{32}$$

This fraction should be represented as a mixed number.

$$\frac{65}{32} = ? \qquad 32\,\overline{)65}\;=2\,\frac{1}{32},\quad 65-64=1$$

$$\frac{65}{32} = 2\,\frac{1}{32}$$

VIII. Division of Mixed Numbers

Division of mixed numbers is similar to multiplication. Convert the mixed numbers to fractions and then proceed as in a regular division problem with fractions.

example: $3\frac{2}{3} \div 2\frac{3}{4} = ?$

$$3\frac{2}{3} = \frac{?}{3} \longrightarrow 3 + \frac{2}{3} = \frac{9}{3} + \frac{2}{3} = \frac{11}{3}$$

$$2\frac{3}{4} = \frac{?}{4} \longrightarrow 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$$

$$\frac{11}{3} \div \left(\frac{11}{4}\right) = \frac{11}{3} \times \frac{4}{11} = \frac{\cancel{11} \times 4}{3 \times \cancel{11}} = \frac{4}{3}$$

$$\begin{array}{r} 1\frac{1}{3} \\ 3\overline{)4} \\ \underline{3} \\ 1 \end{array} \qquad \frac{4}{3} = 1\frac{1}{3}$$

IX. Addition and Subtraction of Mixed Numbers

When <u>adding</u> mixed numbers together add the whole numbers together and add the fractions together. It is <u>not</u> necessary to convert mixed numbers to fractions in order to add them. However, remember to find a common denominator for the fractional parts.

example: $4\frac{3}{8} + 6\frac{1}{12} = ?$

The common denominator is 24. Therefore,

$$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24} \quad \text{and} \quad \frac{1}{12} \times \frac{2}{2} = \frac{2}{24}$$

$$\begin{array}{rcr} 4\frac{3}{8} & = & 4\frac{9}{24} \\ +\ 6\frac{1}{12} & = & \underline{6\frac{2}{24}} \\ ? & & 10\frac{11}{24} \end{array}$$

In a subtraction problem, follow the same procedure, subtracting whole numbers from each other and fractions from each other.

example: $6\frac{2}{5} - 2\frac{1}{7} = ?$

The common denominator is 35.

$$\frac{2}{5} \times \frac{7}{7} = \frac{14}{35} \qquad \frac{1}{7} \times \frac{5}{5} = \frac{5}{35}$$

$$\begin{array}{rcr} 6\frac{2}{5} & = & 6\frac{14}{35} \\ -\ 2\frac{1}{7} & = & 2\frac{5}{35} \\ \hline ? & & 4\frac{9}{35} \end{array}$$

DECIMALS

I. Definition: Decimal notation is a way of representing fractions which have denominators of 10 or a multiple of 10 (100, 1000, 10,000).

The prefix <u>deci</u> means 10. Our number system is of base 10. This means that 10 and multiples of 10 determine the <u>place values</u> in our number system. The following chart labels the place values in our base 10 system.

thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten thousandths	hundred thousandths
1	2	3	4.	5	6	7	8	9

245.62 is read "two hundred and forty-five and sixty two hundredths."

.7543 is read "seven thousand five hundred and forty-three ten thousandths."

Our money system illustrates how a base 10 system works. One cent is $\frac{1}{100}$ of a dollar. It can be written $.01. When 10 one-cent pieces (pennies) are added together, we move into the next place value. This can be represented as $\frac{10}{100}$, $\frac{1}{10}$ of a dollar, or $.10. One hundred pennies equal one dollar, which is written as $1.00.

II. Converting Decimals to Fractions

When converting decimals to fractions you must first figure out which place value represents the digit all the way to the right.

example: 2.541

One is the digit all the way to the right and the corresponding place value is thousandths.

This place value is then used as the denominator (1000). The numerator is simply the entire decimal number, with the decimal point removed (2541).

$$2.541 = \frac{2541}{1000}$$

This number can be converted to a mixed number, and will become $2\frac{541}{1000}$.

Note that the number of places to the right of the decimal point equals the number of zeros in the denominator.

example: $.03 = \frac{3}{100}$ (three hundredths)

example: $.7924 = \frac{7924 \div 4}{10{,}000 \div 4} = \frac{1981}{2500}$

III. Converting Fractions to Decimals

A fraction with the denominator equal to a multiple of 10 can be converted to a decimal very easily. First, count the number of zeros in the denominator. Then copy over the numerator. Starting from the digit all the way to the right, count digits, from right to left up to the number of zeros in the denominator. Place the decimal point to the left of this digit.

example: $\frac{4781}{100} = ?$

There are 2 zeros in the denominator. Count 2 digits from the right - 4781 - and place decimal to the left.

↑↑
21

$$\frac{4781}{100} = 47.81$$

Note: If there are more zeros in the denominator than there are digits in the numerator, add as many zeros as necessary to the left of the numerator.

example: $\frac{34}{10,000} = ?$

There are 4 zeros in the denominator. Since there are only 2 digits in the numerator, add 2 zeros to the left - 34 = 0034. Now count 4 and place the decimal point to the left.

0034 = .0034
↑↑↑↑
4321

A fraction without a multiple of ten as the denominator can be converted to a decimal by dividing the denominator into the numerator. In most cases you will have to add the decimal point to the numerator and add zeros to the right of the decimal point in order to have the division come out evenly. You can add as many zeros as you need to the right of the decimal point. When doing the division, remember to carry the decimal point over the division sign.

example: $\frac{1}{8} = ?$ 1 = 1.000

```
     .125
8  |1.000
    8
    20
    16
     40
     40
      0
```

$\frac{1}{8} = .125$

Sometimes this division will not come out evenly. In such cases either the division continues indefinitely in no specific pattern or the division shows a pattern of repeating digits (one or a series of digits).

Repeating digits are represented by a bar over those digits which repeat.

example: $\frac{2}{3} = .6\overline{6}$

```
    .666
3  |2.000
    1 8
      20
      18
       20
```

example: $\frac{2}{11} = .18\overline{18}$

```
     .1818
11  |2.0000
     1 1
       90
       88
        20
        11
         90
```

The following table lists some commonly used fractions and their decimal representation. Memorizing these conversions should speed up your work.

$\frac{1}{2} = .5$	$\frac{1}{10} = .1$
$\frac{1}{3} = .3\overline{3}$	$\frac{3}{4} = .75$
$\frac{1}{4} = .25$	$\frac{2}{3} = .6\overline{6}$
$\frac{1}{5} = .2$	$\frac{1}{25} = .04$
$\frac{1}{8} = .125$	$\frac{1}{100} = .01$

IV. Adding and Subtracting Decimals

Adding and subtracting decimals follows the same rules as adding and subtracting money.

example: If you have one dollar and you spend 73¢ how much money do you have left?

one dollar = \$1.00
73¢ = .73

$$\begin{array}{r} \$1.00 \\ -\ \underline{.73} \\ .27 \end{array}$$

The answer is .27 or 27¢.

The key to success is lining up the decimal points. Once the decimals are in line, work from right to left down the columns.

example: .532 + .219 + .90 + .0002 = ?

Line each decimal up as follows:

$$\begin{array}{r} .532 \\ .219 \\ .90 \\ +\ \underline{.0002} \\ 1.6512 \end{array}$$

Start from the right-most column and add.

If a whole number is in the problem, remember that you can change it to a decimal by adding a decimal point to the right end of the number.

example: .25 + .017 + 15.1 + 7 = ?

7 = 7.

```
    .25
    .017
  15.1
+  7.
  22.367
```

The same rules apply to subtraction.

example: $.103 - .091 = ?$

```
  .103
- .091
  .012
```

Remember to line up the decimal point and start subtracting from the right-most column.

Zeros added to the right of the last digit in a decimal will not change the value of the decimal. Adding zeros may be helpful in visualizing the computations.

example:

```
  .024          .024
+ .1     --->   .100
   ?            .124
```

example:

```
  .2            .20
- .05    --->  -.05
   ?            .15
```

Note: Zeros to the left of the digits and between the digits and the decimal point are place holders and cannot be removed without changing the value of the fraction. The decimal number .05 does not equal .5 or .005. However, .05 does equal .050 and .0500.

$$.05 \neq .5 \neq .005$$
$$.05 = .050 = .0500$$

When subtracting, "borrowing" one from the column to the left can be done when necessary, the same way it is done when subtracting whole numbers.

example:

```
                        6 1
  .704      .704       .704
- .39       .390       .390
   ?          ?        .314
```

Remember to check subtraction by adding the answer to the amount subtracted. These 2 should equal the first number. To check the answer in the problem above we would add .314 + .390.

```
  .314
+ .390
  .704
```

The result, .704, equals the top number in the problem above, so we see the subtraction was correct.

V. Multiplying Decimals

To multiply two decimal numbers together, begin by multiplying the numbers and ignoring the decimal points. Then count the total number of places (digits) to the right of the decimal point in both original numbers. This will be the number of places after the decimal point in the answer. Starting from the digit all the way to the right, count this number of places to the left, and place the decimal point there.

example: 1.2 x .09 = ?

```
   1.2     ¹12
 x .09 →   09
   ?      108
```

Since there is 1 digit to the right of the first decimal and 2 to the right of the second, we count back 3 digits in the answer.

108 = .108

If there are not enough digits in the answer, add zeros to the left (to fill in the place values).

example: .0002 x .03 = ?

```
   .0002    0002
 x   .03 →    03
     ?         6
```

Since we have to count 6 places, add 5 zeros.

.000006 is the answer.

VI. Dividing Decimals

To divide decimals when the divisor (the number you are dividing by) is not a decimal, carry out the division as usual without removing the decimal point from the dividend (the number being divided). The decimal point is placed in the answer (the quotient) directly above the decimal in the dividend.

example: .4963 ÷ 7 = ?

First set up the problem for division. Place the decimal directly above the decimal in the dividend.

```
     .
 7 ⟌.4963
```

Since 7 divides into 49, not 4, place a zero in the first place to the right of the decimal as a place holder. Now divide, as usual.

```
    .0709
 7 ⟌.4963
     49
     063
      63
       0
```

To divide when the divisor is a decimal, move the decimal point in the divisor to the right end of the number. Now move the decimal point in the dividend the same number of places to the right. Put the decimal for the quotient directly above the newly placed decimal in the dividend.

example: $361.6 \div .08 = ?$

```
                         4520.
.08 ⌐361.60.        08 ⌐36160.
                        32
                         41
                         40
                          16
                          16
                           00
```

Move the decimal in the divisor (.08) two places to the right. Do the same in the dividend (361.6). You will have to add a zero to the right of the 6 in order to move this decimal point 2 places. Place the decimal in the quotient and divide. Do not forget to include the zero between the 2 and decimal point in the answer.

example: $.04745 \div .13 = ?$

```
                         .365
.13  ⌐.04745        13 ⌐4.745
                        3 9
                          84
                          78
                           65
                           65
                            0
```

PERCENT

I. Definition: A percent is a fraction with a denominator of 100. Because they are used so frequently, hundredths have been given this special notation, percent, written %.

example: 20% (20 percent) means 20 hundredths or $\frac{20}{100}$.

325% (325 percent) means 325 hundredths or $\frac{325}{100}$.

.5% (.5 percent) means .5 hundredths or $\frac{.5}{100}$.

II. Changing Percents to Decimals

Because percents are fractions with denominators of 100 (which is a multiple of 10), they can easily be written in decimal notation. Do this by moving the decimal point two places to the left. If no decimal point is given, remember to write one in to the right of the last digit.

example: $5\% = 5.\% = \frac{5}{100} = .05$

$.6\% = \frac{.6}{100} = .006$

$370\% = \frac{370}{100} = 3.70$

Decimals can be converted to percentages by doing the opposite, moving the decimal point 2 places to the right.

example: .71 = 71%

If you have trouble remembering which way to move the decimal point, thinking of your sales tax can be helpful. You can write on your scrap paper, for example:

7% = .07 .07 = 7%

(The tax is 7 cents on every dollar spent). Then, when a percent like .035% comes along, and you need to convert it to a decimal, it becomes easier. Looking at the tax, we see that 7% = .07. The decimal point was moved two places to the left, so we would do the same to .035%.

.035% = .00035

If you had a decimal like .0068, and needed to convert it to a percent, it would become easier by contrasting it to .07 = 7%. Here the decimal point was moved two places to the right, so we would do the same:

.0068 = .68%

Before we can add, subtract, multiply or divide with percents, they must be rewritten in fractional or decimal notation.

Percentages, like fractions, express part of something. 100% represents a whole quantity. 100% of the students in the class means all the students. 100% of any number is itself. Think of 100% as "all."

example: 100% of 5 is 5.
83 is 100% of 83.

The relationship between two numbers is often shown by expressing one as so many percent of the other.

example: 10 is 50% of 20

Think of the word "is" as an equals sign (=) and the word "of" as a multiplication sign (x). Now we can transform the above statement into a math equation.

$$10 \text{ is } 50\% \text{ of } 20$$

$$10 = 50\% \times 20$$

$$10 = \frac{50}{100} \times 20$$

$$10 = \frac{1000}{100}$$

or

$$10 = .50 \text{ of } 20$$

$$10 = .50 \times 20$$

$$10 = 10$$

example: 25% of 8 is 2

Translating that into a math equation we see that:

$$25\% \times 8 = 2$$

$$\frac{25}{100} \times 8 = 2$$

$$\frac{200}{100} = 2$$

or

$$.25 \times 8 = 2.0$$

Using this example we see that a percent (25%) of a base number (8) is a percentage (2) of the base. The standard equation is:

Percent x Base = Percentage

Given values for any two elements of this equation, we can find the third.

example: Find 5% of 250.

We are given the percent and the base and must find the percentage.

$$5\% \times 250 = \text{percentage}$$

$$\frac{5}{100} \times \frac{250}{1} = \frac{1250}{100} = 12.5$$

or

$$.05 \times 250 = 12.5$$

Therefore, 12.5 is 5% of 250.

example: 30 is what percent of 50?

$$30 = \text{percent} \times 50$$

When you are looking for the percent and your equation, like the one above, has a number on either side of the equal sign, take the number that the percent is multiplied by and divide it into the number on the other side. (Or, looking at it another way, move the number to the other side of the equal sign and change the multiplication sign to a division sign.)

$$\frac{30}{50} = \text{percent} \times 50$$

$$\frac{30}{50} = \text{percent}$$

To change $\frac{30}{50}$ to a fraction with a denominator of 100, multiply both the numerator and denominator by 2.

$$\frac{30 \times 2}{50 \times 2} = \frac{60}{100}$$

$$\text{percent} = \frac{60}{100} = 60\%$$

example: 12 is 60% of what number?

In this problem we are asked to find the <u>base</u> number.

$$12 = 60\% \times \text{base}$$

$$12 = \frac{60}{100} \times \text{base}$$

We solve this, like finding the percent in the problem above, by moving $\frac{60}{100}$ to the left side and dividing 12 by it.

$$12 \div \frac{60}{100} = \text{base}$$

Remember that to divide by a fraction you invert the top and bottom and then multiply.

$$12 \div \frac{60}{100} = \text{base}$$

$$\cancel{12}^{1} \times \frac{100}{\cancel{60}_{5}} = \text{base}$$

$$\frac{100}{5} = 20 = \text{base}$$

We use percents every day to describe many situations. Percents are common in word problems too, so some have been included.

example: Forty-five of the 60 employees of Department A attended the annual picnic. What percentage of the department's employees attended?

We are given:

Total number of employees = 60 = base number

Part of total = 45 = percentage

to find the percent.

This problem can be reworded as follows:

45 is what percent of 60?

$$\frac{45}{60} = \text{percent} \times 60$$

$$\frac{45}{60} = \text{percent}$$

Convert $\frac{45}{60}$ to a decimal be dividing the numerator by the denominator.

```
      .75
60 )45.00
    42 0
     3 00
```

Convert the decimal .75 to a percent by moving the decimal point 2 places to the right.

.75 = 75%

III. Percent Increase and Decrease

Percents are commonly used to compare changes in quantity. For example, the couch you want may be reduced by 15% and your car insurance increased by 5%. The changes are expressed as a percentage of the original quantity (base number). We can also see from such comparisons that a $5 increase in a $10 book (to $15) is a lot more significant than a $5 increase in a $100 bicycle (to $105).

Using our knowledge of percents, we can calculate that:

$5 is 50% of $10, but

$5 is 5% of $100.

The book's price increased by 50% while the bicycle's price increased by only 5%.

The new prices of the book and bicycle are found by adding the original price plus the increase. Therefore,

New amount = Original + Increase

15 =	10	+	5
105 =	100	+	5

Likewise, when a price is reduced, the new amount is found by subtracting the decrease from the original price.

New amount = Original - Decrease

Let's look at other examples.

Percent Decrease

A couch originally selling for $480 is now reduced by 15%. How much is the couch selling for on sale?

We are looking for the new price. We know that: New price = Original - Decrease. Since we have the original price, we must first find the decrease.

15% of $480 is the decrease

.15 x 480 = 72, or

$\frac{15}{10\emptyset} \times 48\emptyset$ = decrease

$\frac{720}{10}$ = decrease

Going back to our formula:

New price = Original - Decrease

New price = $480 - 72

New price = $408

We can write both steps in one equation:

New price = Original - (% x Original)

New price = \$480 - $(\frac{15}{100} \times 480)$

When working with multiplication or division and addition or subtraction in the same equation, always do the multiplication or division first.

Percent Increase

Your car insurance costs $220 each six months. With your last bill, you were notified of a 5% increase in cost. How much will your next insurance bill be?

We are given the original number and the percent change. First, find the amount of the increase:

5% of $220 is the increase

5% x $220 = increase

$\frac{5}{10\not{0}} \times 22\not{0}$ = increase

$\frac{110}{10} = 11$ = increase

Then use the formula:

New amount = Original + Increase

New amount = 220 + 11

New amount = 231

This can be done in one equation as follows:

New amount = Original + (% x Original)

New amount = 220 + (5% x 220)

New amount = 220 + ($\frac{5}{100}$ x 220)

231 = 220 + 11

Note: It is important to realize that the change is expressed as a percentage of the original quantity, not of the new value.

220 increased by 5% is 231. The change is 11 and 11 is 5% of 220. 11 is not 5% of 231.

Likewise, from our first problem, the couch was reduced by 15%. $480 decreased by 15% is $408. The decrease is $72. 72 is 15% of 480. 72 is not 15% of 408.

Another way to look at percent increase or decrease is as follows:

The original price of the couch ($480) is 100%. When the couch is reduced by 15%, the new price will be 100% - 15% or 85%. If you look at the problem this way, the new price is 85% of the original price.

New price = 85% of $480

New price = .85 x 480

New price = $408

Likewise, your original car insurance ($220) is 100%. When it increases 5%, your new insurance will be 100% + 5%, or 105%. The new insurance is 105% of the original insurance.

New insurance = 105% of $220

New insurance = 1.05 x 220

New insurance = $231

When working with prices, the terms "mark-up" and "mark-down" can be used to describe percent increase and decrease. Mark-up means the price is increased by a given percent, while mark-down means it has been decreased by a given percent.

example: A portable radio is marked down 20% to a cost of $64. What was the original cost of the radio?

We solve this like a percent decrease problem. We don't know the original price, but we <u>do</u> know the <u>new</u> price and the percent decrease. If the original price is 100%, the new price is 100% - 20%, or 80% of the original. This problem can be rewritten: The new price is 80% of the original price.

$64 = 80\% \times \text{original}$

$64 = \frac{80}{100} \times \text{original}$

$64 \div \frac{80}{100} = \text{original}$

$\overset{8}{\cancel{6}4} \times \frac{10\cancel{0}}{8\cancel{0}} = \$80 = \text{original price, or}$

$64 \div .80 = 80$

It is possible to figure out the percent increase or decrease if the original amount and the new amount are given.

example: Joan's electric bill was $56 last month and $60 this month. By what percent did her bill increase?

New amount = Original + Increase

$60 = $56 + increase

60 - 56 = increase

$4 = increase

$$\text{increase} = \% \times \text{original}$$

$$4 = \% \times \$56$$

$$\frac{\cancel{4}1}{\cancel{5}\cancel{6}14} = \%$$

$$\frac{1}{14} = \%$$

```
       .0714
14 ⟌1.0000  = .07%
      98
      20
      14
       60
       56
        4
```

There is another way to do these type of problems, which are often found on tables on exams, that will always work, and is easier for some people. To determine the percent increase or decrease, FIRST FIND THE DIFFERENCE BETWEEN THE TWO NUMBERS BEING CONSIDERED, AND THEN DIVIDE THIS DIFFERENCE BY THE ORIGINAL NUMBER, THE NUMBER THAT CHRONOLOGICALLY CAME FIRST. In this case, the difference is $4. This difference should be divided by the original number, last month's $56 figure.

$$\frac{4}{56} = .07 = 7\% \text{ increase}$$

These type of questions are reviewed in detail, with plenty of practice questions, in Booklet #3, Understanding and Interpreting Tabular Material.

RATIOS

A ratio is a comparison of one number to another. A ratio between two numbers is expressed by putting a colon (:) between them like this:

Ratio of women to men in the office is 1 to 2.

women in office: # men in office = 1:2

Ratios can be written as fractions:

women in office: # men in office = 1:2 = $\frac{1}{2}$

Ratios can be reduced to simplest form by writing them as fractions and simplifying. Given the ratio as 3:6 we can re-write it as $\frac{3}{6}$ and then reduce it to $\frac{1}{2}$.

example: Reduce 5:10.

$$5:10 = \frac{5}{10} = \frac{\not{5}1}{\not{1}\not{0}2} = \frac{1}{2}$$

example: Reduce 7:28.

$$7:28 = \frac{7}{28} = \frac{\not{7}1}{\not{2}\not{8}4} = \frac{1}{4}$$

Sometimes the term "ratio" will be used in a word problem, but often the language will suggest a comparison and you must recognize it as a ratio problem. Look for key words such as "compared to," "is to," "out of," "relationship between." Working with ratios involves working with fractions:

1 woman <u>out of</u> 20 = 1:20 or $\frac{1}{20}$

3 <u>is to</u> 9 = 3:9 or $\frac{3}{9}$

When asked to work with ratios, you are usually given one comparison or ratio (or fraction) and asked to find an equivalent one.

example: There are 340 employees in an office. One out of every 20 workers will take a vacation in June. How many workers will take a vacation in June?

You are given the ratio 1 to 20 or 1:20, $\frac{1}{20}$. You are asked how many of the 340 workers will take a vacation in June. To set up an equivalent fraction, first note what the comparison is and as a safeguard, write it out next to your work. (The line of the fraction can be substituted for any of the key words listed above.) The comparison is <u>workers taking a vacation</u> to <u>all workers</u>.

$$\text{(compared to)} \ \frac{\text{workers on vacation}}{\text{all workers}} : \frac{1}{20} = \frac{?}{340}$$

Another way to read it would be to say, "1 is to 20 as what is to 340?"

1:20 = ?:340

$$\frac{1}{20} \text{ of } 340 = \frac{1}{20} \times 340 = \frac{34\not{0}}{2\not{0}} = 17, \text{ or}$$

$$\frac{1}{20} = .05 \longrightarrow .05 \times 340 = 17$$

example: How far will a plane travel in 15 hours if it travels 1500 miles in 5 hours and continues at the same rate of speed?

The comparison is the number of miles to the number of hours.

$$\frac{\text{miles}}{\text{hours}} = \frac{1500}{5} = \frac{?}{5}$$

$$\frac{\text{miles}}{\text{hours}} = \frac{1500}{5} = \frac{?}{15}$$

We are looking for a number that has the same relationship to 15 that 1500 has to 5. Because we are working with equivalent fractions, this number will also have the same relationship to 1500 that 5 has to 15. Using this second piece of information, we can find the missing number.

We know that 3 x $\underline{5}$ = $\underline{15}$.

We multiply 5 by 3 to get 15.

The missing number is in the same relationship to 1500 as 5 is to 15. Therefore:

$$3 \times 1500 = ?$$

$$3 \times 1500 = 4500$$

In review we see that:

$$\frac{3 \times 1500}{3 \times 5} = \frac{4500}{15}$$

When the relationships between the numbers is not as familiar, we follow the procedure shown below:

$$\frac{1500}{5} = \frac{?}{15}$$

1. First cross-multiply.

$$\frac{1500}{5} \searrow \frac{?}{15} \longrightarrow \quad 1500 \times 15 = 22{,}500$$

2. Now divide this total by the third number, 5 in this case.

$$22{,}500 \div 5 = 4500$$

3. Then check by comparing the fractions (reducing will help in checking the answers).

$$\frac{\overset{300}{\cancel{1500}}}{\cancel{5}\,1} = \frac{\overset{300}{\cancel{4500}}}{\cancel{15}\,1}$$

example: $\frac{3}{7} = \frac{2}{14}$

In this problem, cross-multiply 3 x 14 to get 42. Then divide 42 by the third number, in this case 2, to get 21.

$42 \div 2 = 21$

Now check for equivalence:

$\frac{\not{3}1}{\not{2}\not{1}7} = \frac{\not{2}1}{\not{1}\not{4}7}$

$\frac{1}{7} = \frac{1}{7}$

The ratios we have worked with so far compared a "part" to a "whole." In this next section we will work with ratios which compare parts to parts, as well as parts to the whole.

It is important to determine which type of ratio you are working with in order to set up the correct equation.

example: For every five permanent workers in the office, there is one provisional employee. In an office with 60 employees, how many are provisional?

We are given a comparison of permanent workers to provisional workers, but the question asks you to compare provisional workers to all workers in the office.

We are comparing:

$$\frac{\text{provisional workers}}{\text{permanent workers}} = \frac{1}{5}$$

We want to compare:

$$\frac{\text{provisional workers}}{\text{all workers}}$$

Since there are 60 workers in the office all together, think of $\frac{1}{5}$ as the fraction, reduced to lowest terms, which represents 60 divided into parts in a relationship of 1:5. Then the total number of workers in the office is represented by adding these parts:

provision workers + permanent workers =

1 + 5 = 6

Therefore:

$$\frac{\text{provisional workers}}{\text{all workers}} = \frac{1}{5+1} = \frac{1}{6}$$

We can set this equal to:

$$\frac{1}{6} = \frac{?}{60}$$

Now follow the usual procedure to solve:

1. First cross-multiply.

$$\frac{1}{6} = \frac{?}{60} \longrightarrow 1 \times 60 = 60$$

2. Then divide by the third number.

$$60 \div 6 = 10$$

3. Check by reducing.

$$\frac{1}{6} = \frac{\not{1}\not{0}1}{\not{6}\not{0}6}$$

Another way to do this is to remember that, for this type of ratio problem, it can be solved by always adding the "parts" involved, in this case 1 + 5, and dividing the resulting number, 6, into the total of people given, 60. This will give you the value of <u>each part</u>.

$$60 \div 6 = 10$$

If the problem was slightly changed, and there were 4 permanent employees for every 2 provisional, adding them you'd get 6; dividing it into 60 you'd get 10, but that means <u>each part</u> was worth 10. Since the ratio is now 2 provisional workers to 4 permanent, and now 2 x 10 = 20, there would be 20 provisional workers. Booklet #2, Arithmetic Reasoning, has more examples of this type of question.

Let's re-word the problem again:

> For every five permanent workers in the office there is one provisional employee. If there are 50 permanent workers, how many total employees in the office are there?

Again, we have to translate a part-to-part ratio to a part-to-whole ratio. If there are five permanent workers to one provisional worker $\left(\frac{5}{1}\right)$ then what is the ratio of permanent workers to all employees? Add the parts (permanent + temporary = 5 + 1 = 6) to see that 5 out of 6 $\left(\frac{5}{6}\right)$ employees in the office are permanent.

The comparison we are asked to draw is permanent workers to total employees:

$$\frac{\text{permanent}}{\text{total}} \qquad \frac{50}{?} = \frac{5}{6}$$

Cross-multiply $\frac{50}{?} = \frac{5}{6} \rightarrow 50 \times 6 = 300$, and then divide by 5 $\rightarrow 300 \div 5 = 60$ to reach the answer: 60.

Or, you could say, "What number is in the same relationship to 50 as 5 is to 6," and figure it out by determining what number 50 is $\frac{5}{6}$ of.

$$50 \div \frac{5}{6} = 50 \times \frac{6}{5} = \frac{300}{5} = 60$$

There is a lot of work with ratios, percents, decimals, and fractions in Booklets 2 and 3, Arithmetic Reasoning and Understanding and Interpreting Tabular Material. This booklet is intended to give you a refresher of the basics, so you'll be able to do the word problems and tabular questions more easily.

GOOD LUCK!