



Examination Preparation Booklet

Arithmetic Reasoning

Booklet No. 2



CIVIL SERVICE EMPLOYEES ASSOCIATION, INC.
LOCAL 1000, AFSCME, AFL-CIO
Danny Donohue, President

Booklet #2

Arithmetic Reasoning

The CSEA Examination Preparation Booklet Series is designed to help members prepare for New York State and local government civil service examinations. This booklet is designed for practice purposes only and its content may not conform to that of any particular civil service examination.

Copyright, Reprinted March 1998
Not To Be Reproduced Without Permission

ARITHMETIC REASONING

For most people, the math sections of examinations are the most difficult. This section is one of the least popular, as it consists solely of mathematical word problems. Yet we've found that people can dramatically improve their scores by practicing with word problems before the exam, and consciously cultivating good problem solving habits.

There are usually fifteen Arithmetic Reasoning questions. Most are of moderate difficulty, a few are pretty easy, and two or three are sometimes very tricky. If you get stuck on one of the more difficult questions, you can put a check mark next to it and come back to it later, if there's time. On most exams, people don't usually run out of time, just patience. This is understandable, but points given for correctly answering one or two of these difficult questions may sometimes make a difference in one's place on a promotion list, so it's good to go back and try different approaches to solving a problem, rather than just guessing.

We suggest you check the answer key after every three questions. (Try to be sure to spend enough time on each question. Don't just give up quickly and turn to the answer key, as you won't get as much out of the process if you do this). If you've missed any questions, consult the Self Study Guide and go through the explanation thoroughly before you continue on to the next question. This way you will gain practice and confidence as you go from question to question. (Don't worry if you don't do very well at first. It's been a long time since most people answered these types of questions, if they ever did). We also suggest you use the Diagnostic Worksheet for each question you missed, to gain insight into areas of problem solving you may need to work on. If your exam won't allow you to use a calculator, it's a good idea to practice these questions without a calculator to increase speed, accuracy and confidence. Remember, these questions are not easy for most people. They may seem really difficult at first and some may seem impossible. If, however, a real effort is made to practice and learn from mistakes, scores in this area can improve considerably. We suggest you do these problems again a week before the exam.

Good luck!

ARITHMETIC REASONING

1. If Jean's weekly income doubled she would be making \$120 a week more than Barbara. Jean's weekly income is \$80 more than half of Betty's. Betty makes \$200 a week.

How much does Barbara make?

- | | |
|----------|----------|
| a. \$180 | c. \$240 |
| b. \$200 | d. \$360 |

2. A conference with 3600 participants gathers in Albany. One of every twelve people attending the conference who have ordered meals has special dietary needs. Half of those attending the conference signed up for meals. How many have special dietary needs?

- | | |
|--------|--------|
| a. 266 | c. 150 |
| b. 133 | d. 300 |

3. It costs \$360 for Office X's service contract with a typewriter company to service 18 typewriters for six months. At this same rate, how much would it cost Office X to service six typewriters for three months?

- | | |
|---------|---------|
| a. \$80 | c. \$90 |
| b. \$75 | d. \$60 |

4. In December, an office spent \$480, or 15% of its non-personnel expenses that month, for postage. What were its total non-personnel expenses for December?

- | | |
|-----------|-----------|
| a. \$3200 | c. \$5520 |
| b. \$552 | d. \$7200 |

5. Catherine bought an equal number of \$11.00, \$9.00, and \$8.00 tickets for a concert. She spent \$196 for all of the tickets. How many of each did she buy?

- | | |
|------|--|
| a. 6 | c. 8 |
| b. 7 | d. Cannot be determined from information given |

6. Agency Y employs 13,800 people. Of these, 42% are male, and 50% of the males are age 30 or younger. How many males are there in Agency Y who are older than 30?

- | | |
|---------|---------|
| a. 5796 | c. 3471 |
| b. 2898 | d. 2910 |

20. Agency Y served 187,565 people in 1981. If the agency served 210,515 people in 1982, this reflected an increase of:
- a. 19.10%
 - b. 15.6%
 - c. 12.2%
 - d. 10.9%
21. The number of people attending a weekly training program in the month of January averaged 116 people. If there were 105 people attending the first week, 106 the second, and 125 the third, how many people attended the fourth week?
- a. 118
 - b. 128
 - c. 130
 - d. 124
22. It takes 16 typists 11 days to complete a project. How long would it take 10 typists, if they worked at the same rate, to complete the same project?
- a. 17.6 days
 - b. 6.8 days
 - c. 6.9 days
 - d. 18.4 days
23. If the sum of two numbers is 280, and their ratio is 7:3, then the smaller number is:
- a. 28
 - b. 84
 - c. 56
 - d. 196
24. The population of Metropolis county in 1982 is 130% of its population in 1972. The population in 1972 was 145,000. What was the population in 1982?
- a. 196,425
 - b. 174,612
 - c. 111,539
 - d. 188,500
25. A car travels 50 miles an hour, and a plane travels 10 miles a minute. How far will the car travel when the plane travels 500 miles?
- a. 50.4 miles
 - b. 37.5 miles
 - c. 41.6 miles
 - d. 39.7 miles
26. In a university with 2000 students, the student-faculty ratio is 16:1. If 18% of the faculty have completed some of their own study at the university, approximately how many have not?
- a. 119
 - b. 127
 - c. 23
 - d. 103

ANSWER KEY

ARITHMETIC REASONING

- | | |
|-------|-------|
| 1. c | 21. b |
| 2. c | 22. a |
| 3. d | 23. b |
| 4. a | 24. d |
| 5. b | 25. c |
| 6. b | 26. d |
| 7. d | 27. c |
| 8. c | 28. b |
| 9. a | 29. a |
| 10. d | 30. a |
| 11. b | 31. c |
| 12. a | 32. b |
| 13. c | 33. d |
| 14. a | 34. d |
| 15. d | 35. c |
| 16. b | 36. a |
| 17. c | 37. a |
| 18. a | 38. b |
| 19. a | 39. c |
| 20. c | 40. d |

ARITHMETIC REASONING

For each question you missed, go through the checklist below and place the number of the question missed next to the trait exhibited. This exercise should give you insight into problem solving behaviors that may need work.

Question Number(s)

Trait Exhibited

1. I had little confidence I could solve the problem
2. I couldn't solve the problem, so I gave up
3. I made a careless error
4. I jumped to an incorrect conclusion
5. I misinterpreted the question
6. I "followed a hunch" without checking it through
7. I didn't step back and evaluate the reasonableness of my solution
8. I worked mechanically because I knew it was hopeless
9. I didn't check all of my work
10. I didn't try to visualize the problem
11. I didn't break the problem down into more easily understandable parts
12. I didn't learn from previous problems
13. I tried to solve the problem with out realizing that my understanding of a section of the problem was vague
14. I was inconsistent in my interpretation of words or operations
15. I was falsely reassured because the answer I got was one of the choices, so I didn't check my work

BOOKLET #2

SELF STUDY GUIDE

ARITHMETIC REASONING

SELF STUDY GUIDE

ARITHMETIC REASONING

You should consult this guide whenever you miss a question or aren't sure why you got the answer you did.

You shouldn't get discouraged if you seem totally lost at first. With practice you will improve. In many cases these questions require using methods you may not have used in years, if ever. We have tested this guide with many people, however, and all of them have been able to improve their ability to answer word problems by conscientiously using it. We don't mean to suggest that sometimes it won't be hard work – you may need to re-work and re-read some of the problems many times before they make sense. You will get out of this guide the fruits of whatever effort you put in, and perseverance in problem solving is always critical.

No knowledge of advanced math is required, however, and we have kept our explanations free of jargon and intimidating formulas. Basically, what you need is a knowledge of basic math and perseverance. In explaining the answers, we briefly review working with fractions, percents and ratios. If you feel you need a more thorough review of these, you can order Booklet Number One in the series, Basic Mathematics, or you can consult one of the large number of basic math textbooks that exist.

It's also important to remember that there are often many ways to do a particular problem. We are presenting methods that are the easiest for most people. If you have a different approach, and you consistently get the right answer using it, there's certainly no need to change.

Good luck!

ARITHMETIC REASONING

1. The answer is C. This kind of question is difficult unless you break it down into parts, and solve it step by step. It's only in the next to last sentence that we're actually given someone's salary. We're told that Betty makes \$200 a week. The question asks for Barbara's salary, but it becomes clear after careful reading that we can't find Barbara's salary unless we find Jean's. Jean's weekly income is \$80 more than half of Betty's.

Jean's salary is \$80 more than half of \$200, so Jean's salary is \$80 more than \$100, or \$180. The first sentence tells us that if Jean's weekly income doubled she would be making \$120 a week more than Barbara. Two times Jean's salary of \$180 would be \$120 a week more than Barbara's salary. So, two times \$180, which equals \$360, would be \$120 a week more than Barbara's salary. \$360 is \$120 more than what number? $\$360 - \$120 = \$240$, Choice C.

2. Choice (C) is the answer: It's important in a question like this to identify and break down the information you are given. From the question, we know:

1. 3600 people are attending the conference
2. 1/2 of the 3600 people have ordered meals
3. One out of 12 (or 1/12) of those who ordered meals has special dietary needs

It's important to remember that in order to solve this we must keep in mind that only half of the attendees have ordered meals. So half of the 3600 people, or 1800, have ordered meals. Of these, one out of every twelve has special dietary needs. So 1/12 of the 1800 people signed up for meals have special dietary needs. There are many ways to solve this problem, from this point on. One way is to simply multiply 1800 by 1/12 to find the answer. (To multiply fractions, multiply the numerators by each other, and the denominators by each other.

$$\frac{1}{12} \times 1800 = \frac{1}{12} \times \frac{1800}{1} = \frac{1800}{12} = 150$$

So, 150 of those who have ordered meals have special dietary needs, Choice (C). Or you could have used decimals. 1/12 is expressed in decimal form as .0833 (to find the decimal form of a fraction, divide the numerator, the top number, by the denominator, the bottom number). $1800 \times 1/12 = 1800 \times .0833 = 149.94 = 150$ people. (It comes out a little unevenly because the decimal has been rounded off.) You also could have set up a ratio, comparing those with special needs who ordered meals to all who ordered meals.

A Review of Ratios

Ratios are intimidating for many people in an exam setting. Yet, we use ratios in "real life" – inches to miles on a map, or the ratio of ingredients in recipes in cooking. A ratio shows the relationship between two numbers. In this case, it shows the relationship between those who have dietary restrictions and ordered meals to all those who ordered meals. One out of every twelve people who have ordered meals has special dietary needs, so we need to examine the relationship between the numbers one and twelve, and apply it to the 1800 people who have ordered meals.

$$\frac{\text{Special dietary needs}}{\text{All who ordered meals}} \text{ as } \frac{1}{12} \text{ as } \frac{\text{what number?}}{1800}$$

One way to do this is to cross multiply. To cross multiply, we multiply the top of one number by the bottom of the other.

$$\frac{1}{12} = \frac{?}{1800} \qquad \frac{1}{12} \begin{array}{c} \swarrow \searrow \\ \nwarrow \swarrow \end{array} \frac{?}{1800}$$

$$\begin{aligned}
 12 \text{ times what number? } & 1 \times 1800 \\
 12 \times ? & = 1800 \\
 ? & = \frac{1800}{12} = 150
 \end{aligned}$$

(You have to divide the 1800 by the 12, because you want to “isolate” the ? on one side, since that will give you the answer. Since the 12 and the ? were being multiplied by each other, the only way you could “free” the ? was to move the 12 over to the other side of the equal sign, by dividing the 1800 by the 12. If you get as far as $12 \times ? = 1800$, and can’t remember whether you should multiply or divide, you can still get the correct answer, because the difference between multiplying and dividing is so large that common sense will tell you which is right. In this case, dividing gives you 150 people, and multiplying gives you 270,000, obviously too big a number. So, in a ratio problem, as long as you set up the relationship between the numbers involved correctly (part is to whole as part is to whole), you should be able to solve it.)

Another way some people do ratios is by remembering that “the product of the means equals the product of the extremes”. This means that when you multiply the “inside” numbers in a ratio problem together, and then multiply the “outside” numbers together, they will always equal each other. In this case, we would set it up like this:

$$1 \text{ is to } 12 \text{ as } ? \text{ is to } 1800$$

The “inside” numbers, 12 and ?, would be multiplied together, $12 \times ?$, and would equal the “outside” numbers that have been multiplied together, 1800 and 1. So

$$\begin{aligned}
 12 \times ? & = 1800 \times 1 \\
 12 \times ? & = 1800 \\
 ? & = \frac{1800}{12} = 150
 \end{aligned}$$

All of this may have seemed totally unnecessary, but it’s important to keep these methods in mind for other questions, when using them may be very helpful. Again, there are many ways to solve math problems. If you use different methods than those in this booklet, and your results are consistently correct, there’s no need to change what you’re doing.

3. The answer is D. This problem is actually easier than it may look at first, and there are a number of ways to do it. One way would be to first determine how much the office is paying per typewriter. We’re told that it costs \$360 to service 18 typewriters for six months. So, for a six month period, it would cost, per typewriter, the total amount, \$360, divided by the total number of typewriters, 18. $\$360 \div 18 = \20 per typewriter. So it costs \$20 to service each typewriter for a six month period. We need to find out how much it would cost to service six typewriters for a three

month period. We know the service cost of each typewriter is \$20 for six months. For three months, it would be half that amount, or \$10 per typewriter. Since we are considering six typewriters, the cost would be six typewriters at \$10 each, for a total of $6 \times \$10 = \60 .

Or we could have said that we're being asked to find the cost of one third of the typewriters (6 is $\frac{1}{3}$ of 18), for half the time (3 months is half of six months). One third of the typewriters would cost $\frac{1}{3}$ as much: $\frac{1}{3} \times \$360 = \120 , and since the time involved is $\frac{1}{2}$ of the total time, $\frac{1}{2}$ of \$120 equals \$60. Or we could have set it up using fractions:

$$360 \times \frac{1}{2} \times \frac{1}{3} = \frac{360}{6} = 60$$

4. The answer is A. Many people miss this question, because they aren't quite sure what to do with the 15% figure they're given. We are told that \$480 is 15% of the office's non-personnel expenses for December. So in order to find the answer we need to know what number 480 is 15% of. So, we are asking "480 is 15% of what number?" To find this, we divide 480 by 15%.

$$480 \div 15\% = 480 \div .15 = 3200.$$

It's always a good idea to go back to the problem and check our answer to see if it makes sense. Is 480 15% of 3200? If we divide 480 by 3200, we get $.15 = 15\%$. So 480 is 15% of 3200, and if we multiply 3200 by .15, we get 480.

Some people aren't sure whether to multiply or divide. Again, because of the difference between the two numbers that result, common sense should tell you. In this case, if you had multiplied, you would have gotten \$72 as an answer, which doesn't make much sense. It's important in these exams to step back and evaluate the reasonableness of your solutions, yet people often fail to do this.

It was also possible in this question to work backwards from the answers given to get the right answer. If you weren't sure how to do it, you could have multiplied each choice by 15%, to see which one was equal to \$480. Choice A, $3200 \times .15 = 480$, would have become the obvious choice. This is a legitimate way to solve these types of problems. Some people select Choice D, because they incorrectly multiply, and then aren't sure where to put the decimal points. (If you get confused, we suggest you use the sales tax to help you remember. For example, a 7% sales tax reflects a tax of \$.07 on every dollar. In the corner of your scrap paper you could write, $7\% = .07$; $.07 = 7\%$, or $8.25\% = .0825$; $.0825 = 8.25\%$. If you do this, when a percent like .0035% comes along on a word problem, you'll be able to convert it to decimal form more easily, especially if you're nervous. Consulting the sales tax example, we'd notice that $7\% = .07$ meant the decimal was moved two places to the left when going from percents to decimals, so .0035% would equal .000035. Or, if we had to convert a decimal like .00046 to a percent, consulting the sale tax, we'd see in the case of $.07 = 7\%$ the decimal was moved two places to the right, so we'd do the same here. .00046 would then equal .046%. Sorry if this was unnecessary, but many people get confused when dealing with decimals and percents on an exam.

5. The answer is B. Many people put D as an answer, thinking that it's not possible to find the answer from the information given. The key thing to note here is that the question says Catherine bought an equal number of tickets. That means that the relationship between each of the ticket amounts will always be equal. There won't be more \$11 tickets than \$9.00 or \$8.00 tickets. (It's true that if the question didn't state there were equal amounts of tickets, D would be the correct answer). Since we know there are equal amounts of each ticket, however, we can find the answer by: 1) adding up the cost of the tickets and 2) dividing this figure into the total dollar amount she paid for them. $\$11 + \$9 + \$8 = \28 . She spent a total of \$196, so we can find out how many of each she bought by dividing 196 by 28. $196 \div 28 = 7$. Some people aren't sure how to do this problem at first, but by spending time looking at the problem and "playing with" the possible choices, it becomes clearer.
6. The answer is B. The first thing we need to do is find the number of males in Agency Y. Agency Y employs 13,800 people, 42% of whom are male. So we would multiply 13,800 by 42% to find the number of males. $13,800 \times 42\% = 13,800 \times .42 = 5796$ males in Agency Y. Of these, 50% are age 30 or younger. So half of the 5,796 males will be older than 30. $5,796 \div 2 = 2898$ males older than 30.
7. The answer is D. There are many ways you could solve this problem. This is another ratio problem (see question #2). We're told a machine can collate 126 400 page books in 14 days. We need to find how many books it could collate in 30 days. Since it collates at the same rate, the answer will be in the same proportion as 126 is to 14. So, $\frac{?}{30} = \frac{126}{14}$. There are many ways to do this.

One is to first find the relationship between 14 and 126. 126 is nine times 14, so the answer will be 9 times 30, or 270. Or, you could set up a ratio and cross-multiply (see #2).

$$\frac{?}{30} = \frac{126}{14}$$

$$\frac{?}{30} \begin{array}{c} \swarrow \searrow \\ \nwarrow \swarrow \end{array} \frac{126}{14}$$

$$14 \times ? = 126 \times 30$$

$$14 \times ? = 3780$$

$$? = 3780 \div 14$$

$$? = 270$$

Or, you could have used “the product of the means equals the product of the extremes.” (See Question #2)

$$14 : 126 \text{ as } 30 : ?$$

$$14 \times ? = 126 \times 30$$

$$14 \times ? = 3780$$

$$? = \frac{3780}{14}$$

$$? = 270$$

8. The answer is C. Many people miss this question, because it’s more difficult than it looks. Many will simply subtract \$360 from \$840, and get \$480, Choice A. Yet, if we go back through the problem and re-read it, we’ll see that Choice A can’t be the correct answer. We know the typewriter and dictation machine together cost \$840. We’re also told that the typewriter cost \$360 more than the dictation machine. So, to find the cost of the typewriter, and to check Choice A, we should be able to add \$360 to Choice A’s \$480 to find the cost of the typewriter. $\$360 + \$480 = \$840$. So, according to Choice A, the typewriter costs \$840. The total cost of the typewriter and dictation machine then becomes $\$360 + \$840 = \$1200$. But the question states the total of both was \$840, so we know something is wrong. We’re looking for an answer that will total \$840 for the two objects, and that will also show that the typewriter is \$360 more. In Choice A, the typewriter comes out to be \$360 more, but the total is not \$840. There are several ways you can do this problem. A very legitimate way is to “work backwards” from the possible answers. You can take each choice add \$360 to it, and see if they total \$840. If you do this, it becomes apparent that Choice C is the answer. $\$240 + \$360 = \$600$, the cost of the typewriter. Adding the typewriter, \$600, and the dictation machine, \$240, they total \$840. The \$600 typewriter is \$360 more than the \$240 typewriter, so it checks out. This is a perfectly acceptable way to solve this problem.

There are many other ways to solve it. One is to use a little algebra.

$$x + x + 360 = 840$$

$$2x = 840 - 360$$

$$2x = 480$$

$$x = 240$$

You don’t need to know algebra to solve it, though. Another way to solve a problem like this is to take the difference between the total of the two numbers and the difference between them, and then divide by two. This will always give you the smaller number.

$$840 - 360 = 480. \quad 480 \div 2 = 240.$$

This method will always work, as it’s the same mathematical operation used in algebra, without the algebra. Working backwards from the answers given will also always work.

9. The answer is A. This question looks more difficult than it is, primarily because unnecessary information is thrown in. The diameter of the wood is not needed. We know that half the length of the wood is needed for the back of the chair. Because the rest of the problem is given in inches, the easiest approach would be to convert $\frac{3}{4}$ of a yard into inches. $\frac{3}{4}$ yard equals how many inches? $\frac{3}{4}$ yards equal $\frac{27}{36}$, 27 inches. We need to find what half of 27 inches is to find how many inches will be used for pegs. $27 \div 2 = 13\frac{1}{2}$ inches. Since $13\frac{1}{2}$ inches are being used for the back of the chair, $13\frac{1}{2}$ inches are left for the pegs. Since each peg is $\frac{3}{4}$ of an inch, we should divide the total length available, $13\frac{1}{2}$ inches, by $\frac{3}{4}$ of an inch to find how many pegs can be made.

$$13\frac{1}{2} \div \frac{3}{4} = \frac{27}{2} \div \frac{3}{4}$$

(to divide fractions, we invert the second fraction and then multiply)

$$\frac{27}{2} \times \frac{4}{3} = \frac{108}{6} = 18 \text{ pegs}$$

Or, if you hate fractions, you could have converted to decimals and then divided.

10. The answer is D. There are several ways to do a problem like this, that asks you to combine the efforts of two people. One way to do this is to remember the following method. First, invert the two numbers you're given, four hours and seven hours. So these numbers become $\frac{1}{4}$ and $\frac{1}{7}$. Then, add them together. ($\frac{1}{4} + \frac{1}{7}$ do not equal $\frac{1}{11}$, as we have to find a common denominator first). Twenty-eight is a number both four and seven will divide evenly into.

$$\frac{1}{4} = \frac{7}{28} \text{ and } \frac{1}{7} = \frac{4}{28}$$

$$\frac{7}{28} + \frac{4}{28} = \frac{11}{28}$$

The last thing needed is to invert again to find the answer.

$$\frac{28}{11} = 2.545$$

If you remember this method, invert, add together, and invert again, you will always be able to answer this type of question. The problem can also be solved algebraically. Let x be the time it takes. Robin can wallpaper a room in 4 hours, and Susan can wallpaper a room in 7 hours. In x

hours, Robin does $\frac{x}{4}$ part of the work, and Susan does $\frac{x}{7}$ part of the work.

Together they do the complete job.

$$\frac{x}{4} + \frac{x}{7} = 1$$

(Multiply the equation by 28 to get rid of the fractions)

$$\begin{array}{rcl} 7x + 4x & = & 28 \\ 11x & = & 28 \\ x & = & \frac{28}{11} = 2.545 \end{array}$$

Sometimes it's also possible to estimate an answer in these types of questions. When that isn't possible, it's good to use the first method or the algebraic method, as they always work with this type of work problem.

11. The answer is B. Most people find these types of questions irritating, not only because they bring back bad memories, but also because of their basic irrationality – who ever jogs, (or wallpapers, or builds chairs) at exactly the same rate every time? Nevertheless, it's important to know how to do these. In this problem, we need to first find Mary's speed. We can do this by dividing the distance she traveled (3 miles) by her time (40 minutes). $3 \div 40 = .075$ miles per minute. We know Mary runs for 40 minutes. Alice runs the same distance in 30 minutes, so Mary has 10 minutes of running time left when Alice finishes. To find how much distance Mary has left, multiply her speed, or rate by the time left.

$$.075 \times 10 = .75, \text{ or } 3/4 \text{ of a mile.}$$

12. The answer is A. We first need to determine how much the organization spent on each group of tickets.

50	\$8 tickets	=	50x8	=	\$400
25	\$10 tickets	=	25x10	=	250
25	\$15 tickets	=	25x15	=	375

Since all the \$8 and \$15 tickets were sold at 25% above cost, the money spent on these tickets came back and the organization made a profit of 25%. So the total spent on these two groups of tickets was $\$400 + \$375 = \$775$.

We need to find the 25% profit on these.

$$25\% \text{ of } 775 = .25 \times 775 = \$193.75.$$

\$193.75 is the profit from the \$8 and \$15 group of tickets. Of the 25 ten dollar tickets, three were unsold. Since each ticket was worth \$10, and there were three unsold, \$30 was spent by the organization in buying those tickets that did not come back. So the \$30 will have to be subtracted from whatever profit is made. We should then find the profit on the 22 ten dollar tickets that did sell. $22 \times 10 = \$220$. 25% profit on \$220 = $.25 \times 220 = \$55.00$. There was a \$55 profit on the ten dollar tickets sold. But we need to subtract the \$30 worth of unsold tickets from this. $55 - 30 = \$25$ profit on ten dollar tickets. Remember that the organization made \$193.75 on the other ticket sales. Adding \$25 to \$193.75, the total profit was \$218.75

13. The answer is C. Another percent problem. There are many ways to do this. One way to do this is to ask, \$640 is 80% of what number?

$$\frac{640}{.80} = 800$$

(If you're not sure whether to multiply or divide, you could still figure it out. If you multiply,

you should notice that the answer you get, \$512, is less than \$640, so it couldn't be correct. Another way to do it would have been to set up a ratio.

$$\frac{640}{?} = \frac{80}{100} \text{ (80\% is to 100\%)}$$

$$80 \times ? = 640 \times 100, \quad 80 \times ? = 64,000, \quad ? = \frac{64,000}{80}, \quad ? = 800$$

Or, you could work backwards and go through each choice, multiplying by 20% and then subtracting this amount, to see which choice would give you \$640. Choice C is \$800.

$$\$800 \times .20 = 160. \quad \$800 - \$160 = \$640.$$

14. The answer is A. It's first necessary to determine the total amount spent on travel costs in the three days. The total mileage was $145 + 72 + 98 = 315$. The salesperson traveled 315 miles, at 21¢ per mile. So the cost is $315 \times .21 = \$66.15$. We need to find what percent the travel costs were of sales. To do this we divide \$66.15 by \$2300.

$$66.15 \div 2300 = .0287 = 2.87\%.$$

Choice A, 3% is the closest of the four possible choices.

15. The answer is D. This is a tricky question, because many people assume they must set up a ratio, and find the number of students. But the question is asking for the percent of students, not number of students. One out of every eight students takes Latin. The number 650 is irrelevant. All we need to find is what percent 1 is of 8. $1 : 8 = .125 = 12.5\%$. 12.5% is closest to choice D, 13. (They don't need to put a percent sign next to each choice, as the question is asking for an answer in percents.) Once again, this question shows the importance of reading the problem carefully.

16. The answer is B. We first need to determine the total value of the inkind contributions by adding \$14,500 and \$1,200. $14,500 + 1,200 = \$15,700$. Then we need to find what percent of the total of \$102,000 the inkind total of \$15,700 is. We can do this by dividing the inkind contributions by the total budget.

$$\frac{15,700}{102,000} = .1539 = 15.4\%$$

17. The answer is C. We know from question #16 that the program has a budget of \$102,000. We can find the answer by setting up a relationship between the pounds of food distributed to the money budgeted.

$$\frac{\text{lbs. of food}}{\text{money budgeted}} = \frac{250,000}{102,000} = 2.45$$

There were 2.45 pounds of food distributed for each dollar spent.

18. The answer is A. One way to do this is to first find how many cases of jars were purchased. Jars are sold in cases of 12, not individually. If there are 76 quarts of tomatoes, dividing 76 by 12, we get 6.33 cases. Since we can't buy jars individually, we need 7 cases. For 20 pints of jelly, we'll do the same, dividing 20 by 12. $20 \div 12 = 1.66$, so we'll need 2 cases. We'll need a total of 9 cases of jars. The regular price per case of jars is \$4.25. But there is a discount of 15%.

$$15\% \text{ of } 4.25 = .15 \times 4.25 = .637 = .64 \text{ (to the nearest cent).}$$

\$4.25 - the discount of .64 = \$3.61 per case. There are 9 cases that are purchased at \$3.61 per case. So the cost of jars alone is $\$3.61 \times 9 = \32.49 . Next we have to find the cost of the processing. The cannery charges 15¢ per quart. There are 76 quarts at 15¢ per quart, so the cost of processing equals $76 \times .15 = \$11.40$. There are 20 pints that need to be processed at a cost of 25¢ for 2 pints, so the cost of processing will be $20 \times .25/2 = \$2.50$, or $10 \times .25 = \$2.50$. So, the cost of the canning will be the total of the cost of the jars, \$32.49, the cost of processing 76 quarts, \$11.40, and the cost of processing 20 pints, \$2.50. $\$32.49 + 11.40 + 2.50 = \46.39 .

19. The answer is A. This is a different kind of ratio problem. There are several ways to solve it. One way is to first add the parts given in the ratio in the problem. $1 + 2 + 7 = 10$. Then divide this number into the total amount of whatever substance you've been given. In this case, it's 12 ounces of cough medicine. This will give the value of each part. $12 \div 10 = 1.2$. Now, to find how many ounces of each ingredient is used, we would multiply 1.2, which represents one part, by the ratio of each of the ingredients given. Ingredient X is worth 1 part, so ingredient

$$x = 1.2 \times 1 = 1.2 \text{ oz.}$$

Ingredient Y is worth 2 parts, so $1.2 \times 2 = 2.4$ oz. Ingredient z is worth 7 parts, so $1.2 \times 7 = 8.4$ oz. We can check to see if adding them would give us 12 ounces. $1.2 + 2.4 + 8.4 = 12.0$ ounces, so it checks out. The question asks us for the amount of ingredient Y, 2.4 oz, choice A. Or, you could have expressed this algebraically:

$$A + 2A + 7A = 12$$

$$10A = 12$$

$$A = \frac{12}{10}$$

$$A = 1.2$$

$$2A = 2.4$$

Or, you could work backwards from each choice, but in this case it's more work than using the above methods.

20. The answer is C. This is a percent increase question. Percent increase and decrease problems occur very often on the tabular section, and occur on this section as well. We're told the people served by Agency Y increased from 187,565 to 210,515. We need to find the percent increase. **TO FIND PERCENT INCREASE OR DECREASE:** 1) Take the difference between the two numbers being considered, and 2) Divide this difference by the original number, the number that chronologically came first. The difference between 210,515 and 187,565 is 22,950. 22,950 divided by 187,565 (the earlier 1981, figure) equals $.122 = 12.2\%$. If you can remember these two steps, you will always be able to answer this type of question.

21. The answer is B. There are several ways to do this. If, while doing these problems you use different methods, you shouldn't worry as long as you're getting the right answers. There are many ways to approach these problems. One way to do this would be to set up an equation.

$$\frac{105 + 106 + 125 + ?}{4} = 116$$

The average of the three known numbers, and the unknown number equals 116. The above equation shows that if we add the four numbers together, and then divide by 4, we'll get 116. The 4 as a divisor is cumbersome. To get rid of it, we can multiply each side by 4.

$$\begin{aligned} 4 \times \frac{105 + 106 + 125 + ?}{4} &= 116 \times 4 \\ 105 + 106 + 125 + ? &= 464 \\ 336 + ? &= 464 \\ ? &= 464 - 336 \\ ? &= 128 \end{aligned}$$

You can check this if you wish by adding

$$105 + 106 + 125 + 128 = 464.$$

Dividing by 4, we get 116, so it checks out. Or, you could have solved this problem by working backwards, taking each of the possible choices, adding it to the other three numbers, and then dividing by four to see if their average was 116. If you did this, which is a perfectly legitimate way to solve problems of this type, you would also have gotten 128, Choice B.

22. The answer is A. This problem can be solved without a lot of difficulty if the relationships between the workers and their times is kept clearly in mind. If it takes 16 typists 11 days to complete a project, we need to find how long it will take 10 typists working at the same rate. The 10 typists would complete the job $10/16$ as quickly. So we could find the answer by dividing the days it took 16 typists, 11 days, by $10/16$.

$$11 \div \frac{10}{16} = 11 \div \frac{5}{8} = 11 \times \frac{8}{5} = \frac{88}{5} = 17.6$$

days. If you weren't at all sure how to do this, you may have wanted to first use a simpler example, so that you could then visualize what needed to be done. For example, what if the question had read "It takes 4 typists 8 days to complete a project. It would take 2 typists how many days?" You would have figured out that 2 typists would take twice as long, so it would have taken them 16 days. If you examined how you got this answer more carefully, you would be able to derive a method that could be used to solve the question. 2 typists is half of four typists. The 2 typists would complete the job $2/4$, or half as quickly. The number of days it took was 8. You would then divide 8 by $1/2$ to get the answer.

$$8 \div \frac{1}{2} = 8 \times \frac{2}{1} = 16$$

If you weren't sure what you were supposed to do at this point, multiply, divide, or whatever, you had a clue in that you knew the answer was 16. So you would do whatever would give you 16, and that was divide. This is a legitimate way to solve a problem, using a simpler, clearer relationship between two numbers, seeing how the problem would be solved, in that case, and then applying the method to the test question. If you're stuck on how to approach a question, it's a good way to gain insight into how to solve it.

23. The answer is B. This is the same type of ratio problem as Question 19. The first thing we need to do is add the parts of the ratio together. $7 + 3 = 10$. We then divide this into the total of the two numbers, 280. $280 \div 10 = 28$. This means each part is equal to 28. The smaller number will equal 3 parts of 28, and the larger will equal 7 parts of 28.

$$3 \times 28 = 84. \qquad 7 \times 28 = 196.$$

We're asked for the smaller number, 84. We can check this by adding 84 and 196 to see if they equal 280. $84 + 196 = 280$.

24. The answer is D. Another percent problem. The 1982 population of Metropolis county is 130% of its 1972 population, 145,000. So the population will be $130\% \times 145,000$. ($130\% = 1.30$)
 $1.30 \times 145,000 = 188,500$, Choice D. (If you weren't sure whether to multiply or divide, division would have given you a smaller number than the 1972 figure, Choice C, which wouldn't make sense since there was an increase, not a decrease, in population).

25. The answer is C. We know that the plane travels 10 miles a minute, and the car travels 50 miles an hour. To find how far the car will travel when the plane travels 500 miles, we need to first find out how long it will take the plane to travel 500 miles. At 10 miles a minute, the plane will take 500 miles divided by 10 miles a minute,

$$\frac{500 \text{ miles}}{10 \text{ miles/minute}} = 50 \text{ minutes}$$

We need to find how far the car has traveled in 50 minutes. If the car travels 50 miles in 60 minutes, how far will it travel in 50 minutes? We can set up a ratio to find this.

$$\frac{50 \text{ miles}}{60 \text{ minutes}} \quad \text{as} \quad \frac{? \text{ miles}}{50 \text{ minutes}}$$

One way to solve it is to notice that, since this is a ratio problem, these numbers will be in direct proportion to each other.

$$\frac{50}{60} \text{ is } \frac{5}{6}$$

So the answer will be $\frac{5}{6}$ of 50. $\frac{5}{6} \times 50 = \frac{250}{6} = 41.66$.

Or you could cross multiply:

$$\frac{50}{60} = \frac{?}{50}$$

$$60 \times ? = 2500$$

$$? = \frac{2500}{60}$$

$$? = 41.66$$

Or you could use “the means equals the product of the extremes (see Question #2).

$$50 : 60 = ? : 50$$

$$60 \times ? = 2500$$

$$? = \frac{2500}{60}$$

$$? = 41.66$$

26. The answer is D. We first need to find the total number of faculty. We know the ratio of students to faculty is 16:1, and there are 2,000 students. So we can find this by setting up a ratio.

$$\frac{\text{Students}}{\text{Faculty}} \text{ as } \frac{\text{Students}}{\text{Faculty}} \quad \frac{16}{1} = \frac{2000}{?}$$

You can solve from here in a number of ways. One way is to observe that 2000 is 125 times greater than 16, so what we’re trying to find will be 125 times greater than 1. Or, cross multiply:

$$16 \times ? = 2000 \times 1$$

$$16 \times ? = 2000$$

$$? = \frac{2000}{16}$$

$$? = 125$$

Or, use “the product of the means equals the product of the extremes”, (see Question #2).

$$16 : 1 = 2000 : ?$$

$$16 \times ? = 2000 \times 1$$

$$16 \times ? = 2000$$

$$? = \frac{2000}{16}$$

$$? = 125$$

We know there are 125 faculty. If approximately 18% of them studied at the university, then 82% did not. We need to find 82% of 125. $125 \times .82 = 102.5$ (Or you could have multiplied 125 by 18% and then subtracted the result from 125).

27. The answer is C. One of the few good things about multiple choice math questions is that the answer has to be one of the four given. In a problem like this, if you can't figure it out, it's possible to work backwards to get the answer. Most people select Choice A, because they misread and think \$30 is the ten percent profit of the sales price of \$300. But she bought the set for \$300, and the question states that they sell all merchandise at cost plus 10%. If the profit was \$30, the set would have to cost \$300. But this would mean the total cost would be \$300, the cost, plus \$30 (the 10% profit) added on, for a total of \$330, not \$300. So Choice A can't be correct. One way to do this would be to work backwards from each choice given. Choice B states that \$27 is the profit. \$27 is 10% of \$270. Added together they equal \$297, not \$300, so Choice B is incorrect. Choice C is \$27.27. \$27.27 is 10% of \$272.70. $\$272.70 + 27.27 = \299.97 . Since the answer says approximately, this looks like a safe choice. But just in case, if you're not sure, we can check Choice D. \$32.26 is 10% of \$322.60, so we know that they won't add up to \$300. Choice C is the answer. This is a perfectly good way to solve this problem. Arithmetic Reasoning is also testing your resourcefulness at working with numbers, and working backwards if you're stuck is certainly being resourceful. You could also say to yourself, \$300 is 110% (100% is the cost of the item, plus a 10% profit added on) of what number? $300 \div 110\%$ will give us the answer. $300 \div 1.10 = \$272.72$. So we know \$272.72 is the actual cost. The profit will be 10% of this, or \$27.27. Or you could have set up a ratio:

$$\frac{300}{?} = \frac{110}{100} \quad (110\% \text{ is to } 100\%)$$

These methods are quicker than working backwards, so you may want to spend some time studying them.

28. The answer is B. We know that the temperature dropped 8° from midnight until 1 a.m., and before this it dropped at a constant rate. Working backwards from 1 a.m., we can add 8° to the temperature given at 1 a.m., 37° , to find the temperature at midnight. $37 + 8 = 45^\circ$ at midnight. The temperature dropped at a constant rate from six p.m. to midnight. During that time it went from 54° to 45° . This is a drop of 9° . $54 - 45 = 9$, 6 hours elapsed from 6 p.m. to midnight. In 6 hours, the temperature dropped 9° . To find the rate the temperature dropped each hour, we would divide 9 by 6. $9 \div 6 = 1.5$. Since the temperature dropped 1.5° each hour, we can find the temperature for 10 p.m. by subtracting (4 hours times 1.5 degrees), which equals 6° , from 54. $54 - 6 = 48^\circ$, the temperature at 10 p.m. Or we could add two hours times 1.50 degrees, 3° , on to the midnight temperature of 45° , $45 + 3 = 48^\circ$, Choice B.
29. The answer is A. We know that one eighth of a half gallon carton of ice cream has been eaten, and the remainder is divided by three people. The trick to this question, and it's a tricky question, is that they are asking for what percent of a gallon each person gets, not of a half gallon, and there is only a half gallon of ice cream to begin with. So, if $1/8$ of a half gallon has already been eaten, we can find out how much this is by multiplying $1/8$ by $1/2$. $1/8 \times 1/2 = 1/16$. So $1/16$ of a gallon has been eaten.

The remainder is divided by three people. There was a half gallon, but 1/16 has been eaten. That leaves 1/2 - 1/16. $8/16 - 1/16 = 7/16$ left. The remainder, 7/16, is divided by 3 people.

$$\frac{7}{16} \div 3 = \frac{7}{16} \times \frac{1}{3} = \frac{7}{48}$$

So each person gets 7/48 of a gallon. But the answer has to be expressed in percents. 7/48 as a percent is 7 divided by 48 = .1458 = 14.6%, Choice A. Fortunately, few of the exam questions are this tricky. It's always good to re-check your answers, with this type of question.

30. The answer is A. This question looks more difficult than it is. We know the woman got a score of "143 on a scale of 160". This means that out of 160 questions, she got 143 correct. We're asked to convert her score "to a scale of 0-100". All that means is that we're going to convert her score into a percent. (Percents are based on 100.) 143 is what percent of 160? To find this, we divide 143 by 160.

$$143 \div 160 = .893 = 89.4, \text{ Choice A.}$$

31. The answer is C. This is another ratio problem. Since the assessment is at the same rate, we can set up a ratio between the numbers involved. (See Questions 2, 25, 26.)

$$\begin{array}{l} \frac{\text{tax}}{\text{value}} \text{ as } \frac{\text{tax}}{\text{value}} \qquad \frac{88}{28,000} \text{ as } \frac{110}{?} \\ \text{or} \qquad \frac{\text{tax}}{\text{tax}} \text{ as } \frac{\text{value}}{\text{value}} \qquad \frac{88}{110} \text{ as } \frac{28,000}{?} \end{array}$$

You can then solve it. If you cross multiply:

$$\begin{array}{l} \frac{88}{110} = \frac{28,000}{?} \qquad 88 \times ? = 28,000 \times 110 \\ \qquad \qquad \qquad \qquad \qquad \qquad 88 \times ? = 3,080,000 \\ \qquad \qquad \qquad \qquad \qquad \qquad ? = \frac{3,080,000}{88} \\ \qquad \qquad \qquad \qquad \qquad \qquad ? = 35,000 \end{array}$$

Or, you could use "the product of the means equals the product of the extremes".

$$\begin{array}{l} 88 : 110 \text{ as } 28,000 : ? \\ 88 \times ? = 110 \times 28,000 \\ 88 \times ? = 3,080,000 \\ ? = \frac{3,080,000}{88} \\ ? = 35,000 \end{array}$$

32. The answer is B. Here's another work problem (see question 10). For the type of work problem that asks you to combine the efforts of two different people, there are several approaches you can use. One way to do this is to first invert the two numbers you're given, 14 days and 16 days. They become 1/14 and 1/16. Then, add them together. You'll need to find a common denominator to do this.

$$\frac{1}{14} + \frac{1}{16} = \frac{\quad}{112} + \frac{\quad}{112}. \quad \frac{8}{112} + \frac{7}{112} = \frac{15}{112}.$$

Then invert again to find the answer. $\frac{112}{15} = 7.466$ days. Or, you could use algebra, letting x be the time it takes. $\frac{1}{14} + \frac{1}{16} = \frac{1}{x}$ (In x days, Janet does 1/14 of the work, and Anne does 1/16 of the work. Together they do the complete job.)

$$\frac{x}{14} + \frac{x}{16} = 1 \quad (\text{Multiply the equation by 112 to get rid of the fractions.})$$

$$8x + 7x = 112$$

$$15x = 112$$

$$x = 7.466$$

Sometimes it's possible to estimate an answer. When this isn't possible, it's good to use the first method or the algebraic method as they always work with this type of work problem.

33. The answer is D. This is another tricky question. It's important to remember that they got together to settle their accounts. That means they were paying each other back. It's a good idea to break this problem down into parts. A good way to do it is to write each person's name out, with the amount they are paying back or receiving directly under her name. So, Cynthia loaned \$35 to Mary. Since they've gotten together to pay each other back, Cynthia will be getting back \$35 from Mary. So we'll put a +35 under Cynthia's name, and a -35 under Mary's. Cynthia borrowed \$14 from Jean: -14 for Cynthia (she's paying Jean back), +14 for Jean. She also borrowed \$16 from Emily, so -16 for Cynthia, +16 for Emily. Emily owes \$17 to Jean: -17 for Emily, +17 for Jean. Emily owes \$9 to Mary: -9 for Emily, +9 for Mary.

	<u>Cynthia</u>	<u>Jean</u>	<u>Mary</u>	<u>Emily</u>
	+35	+14	-35	+16
	-14	<u>+17</u>	<u>+9</u>	-17
	<u>-16</u>	+31	-26	<u>-9</u>
Total	+5			-10

Emily left with \$10 less than she came with. A lot of work for a point and a half.

34. The answer is D. We know the tiles are square, and that they are 12 inches on each side. To find out how many we'll need, we need to find out how large the floor is. We're told it's 18 feet long and 11 feet wide. Multiplying 18 by 11 will give us the area that needs to be covered. $18 \times 11 = 198$ square feet. Since each tile is exactly a foot on each side, we'll need 198 of them.

35. The answer is C. Another percent problem. We know the car is presently valued at \$3245, and that it's worth 72% of its original cost. To find this, we could say "72% of what number is \$3245?"

$$\begin{aligned} 72\% \times ? &= 3245 \\ .72 \times ? &= 3245 \\ ? &= \frac{3245}{.72} \\ ? &= 4506.94 \end{aligned}$$

Check it: is 3245 72% of 4506.94? $3245 \div 4506.94 = .72 = 72\%$. Or you could have set up a ratio.

$$\frac{3245}{?} = \frac{72}{100} \quad (72\% \text{ is to } 100\%)$$

Or you could have worked backwards, taking 72% of each of the possible answers to see which would give you \$3245.

36. The answer is A. Again, a percent problem. They're very common on this section. We know the sales tax rate is 4%, and the tax on the typewriter was \$13.41. To find the purchase price before the tax was added, we could ask "13.41 is 4% of what number?"
13.41 is .04 of what number?

$$\begin{aligned} 13.41 &= 4\% \times ? \\ .04 \times ? &= 13.41 \\ ? &= \frac{13.41}{.04} \\ ? &= 335.25 \end{aligned}$$

To check it, multiply \$335.25 by 4%. It should equal \$13.41, and it does. Or you could have used a ratio.

$$\frac{13.41}{?} = \frac{4}{100} \quad (4\% \text{ is to } 100\%)$$

You could also have worked backwards, taking 4% of each answer until you got \$13.41.

37. The answer is A. For an interest problem like this one, the interest will equal the rate x principal x time. The time is always expressed as some part of a year. In this problem, the interest will equal $8\% \times 600 \times 1/12$ (one month is 1/12 of a year).

$$.08 \times 600 \times \frac{1}{12} = 48 \times \frac{1}{12} = \frac{48}{12} = \$4.$$

The trick with these types of problems is to remember to express the time in terms of a year. In this case, the 30 days is expressed as one month, or 1/12 of a year.

38. The answer is B. We know the garden is 30 feet by 40 feet. We need to find the perimeter first, to determine how much fencing is needed. Remember, there are four sides. Two will be 40 feet,

and two 30 feet. To find the perimeter, we need to add the four sides. $40 + 40 + 30 + 30 = 140$ feet. It costs \$1.75 per foot for the fencing. So the cost will be $140 \times 1.75 = \$245$. An expensive fence, but the correct answer.

39. The answer is C. Many people miss this question. We need to find what part of her entire year's earnings she earns in April. We know she earns four times as much in April as in each of the other months. One way to do it would be to assign "parts". April would equal 4 parts. Each of the other months would equal one part, so eleven months with one part each would equal eleven parts. The total would then be 15 parts, 4 of which were April's earnings. So April's earnings would equal 4 out of the 15 total parts, or $4/15$. Or, you could have assigned imaginary dollar values to see the relationship more clearly. Imagine she made \$1000 each month. April's earnings would be 4 times that, or \$4000. In the other eleven months she'd make \$11,000. The total would be \$15,000. April's earnings would be \$4000 of the \$15,000, or $4000/15000 = 4/15$. It sometimes helps to bring in "real life" examples to help see relationships more clearly.
40. The answer is D. Another ratio problem. A fitting ending. We're told a train travels 70 mile when a bus travels 50 miles. We need to find how many miles the train will travel when the bus travels 60 miles.

$$\frac{\text{train}}{\text{bus}} \text{ as } \frac{\text{train}}{\text{bus}} \quad \frac{70}{50} \text{ as } \frac{?}{60}$$

One way to do this is to notice that 70 is $7/5$ of 50, so the answer will be $7/5$ of 60. $7/5 \times 60 = 420/5 = 84$. Or, you could cross multiply:

$$\begin{aligned} \frac{70}{50} &= \frac{?}{60} & 50 \times ? &= 70 \times 60 \\ & & 50 \times ? &= 4200 \\ & & ? &= \frac{4200}{50} \\ & & ? &= 84 \end{aligned}$$

Or, you could use "the product of the means equals the product of the extremes". (See Question #2)

$$\begin{aligned} 70:50 \text{ as } ? : 60 \\ 50 \times ? &= 70 \times 60 \\ 50 \times ? &= 4200 \\ ? &= \frac{4200}{50} \\ ? &= 84 \end{aligned}$$

We hope this booklet has helped you improve your ability to solve word problems. There are an infinite variety of word problems available. We've tried to expose you to the most common types of reasoning processes required to solve these problems. It's these processes that are critical, as they can be applied to most word problems. By practicing and sharpening your skills, your ability to solve these kinds of problems should improve.